# Continuity

A function is **continuous at a number**  if:

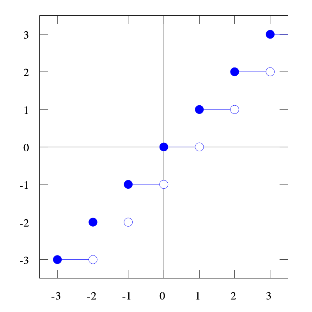
1. exists.
2. exists.
3. .

# File:Function-1 x.svgDiscontinuity

A function is **discontinuous at**  if it is **not continuous at** . You can also phrase this as “the function has a **discontinuity at** .”

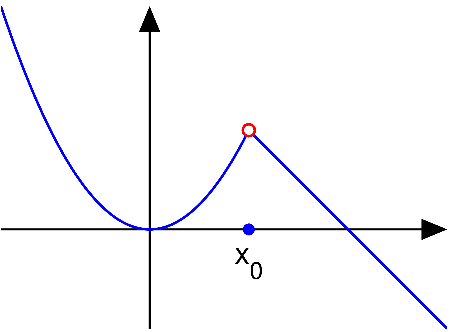
At , is the graph to the right continuous?

To the right is an example of **infinite discontinuity** because there’s an asymptote that comes from either positive or negative infinity.



The floor function is to the left. Every time reaches a new integer, “jumps” to a new value without any continuity in-between. This is called **jump continuity**.

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To the right, notice that at there’s an open circle (○) and a closed circle (●). This means that the open circle is , but the blue circle is the actual value of the function, .

Since , the function to the right is discontinuous at . This is a **removable discontinuity**.

# Continuous from the Side

A function is **continuous from the** **right** if: .

A function is **continuous from the** **left** if: .

# Continuous on an Interval

A function is **continuous on an interval** if the function is continuous at each number in the interval.

If functions and are continuous at , and is a constant, then the following are also continuous at :

**Polynomials** (e.g., ) are **continuous on all real numbers** (), because polynomials don’t have holes anywhere across all the real numbers. Thus, polynomials are continuous on the interval of all real numbers, or .

**Rational functions** (fractions of polynomials), **root functions** (square root, cube root, etc.), and **trigonometric functions** are **continuous across their entire domain**. They do not all have the same domain, though.

# Continuity with Composite Functions

If is continuous at and , then, substituting for , . This is true because, for to be continuous at , rule three says:

1. .

If is continuous at and is continuous at , then the composite function is continuous at .

# The Intermediate Value Theorem

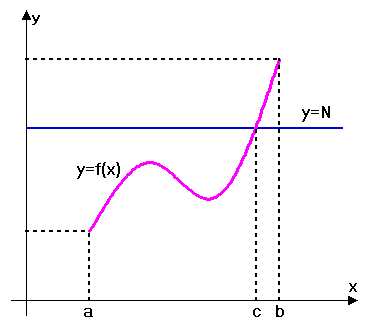
Suppose is continuous on . For any number between and , there is a number c such that . See the graph to the right for illustration.

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